Transmission Line Models for Negative Refractive Index Media and Associated Implementations Without Excess Resonators

George V. Eleftheriades, Omar Siddiqui, and Ashwin K. Iyer

***Abstract*—Recently, three-dimensional composite periodic media comprising split-ring resonators (SRR) and thin wires have been shown to exhibit a negative refractive index in the frequency range around the SRR resonance. In this letter, we propose transmission line models for studying and interpreting the electromagnetic propagation behavior of such materials. Based on these equivalent transmission line models, we show that by periodically loading a network of transmission lines with series capacitors and shunt inductors, a negative refractive index medium can be synthesized *without* excess resonators, thus leading to wideband behavior. These proposed media have tailorable properties over a broad frequency range. Moreover, they are completely planar, frequency scalable, more compact, and easier to implement for RF/microwave circuit applications than their SRR/wire counterparts.**

***Index Terms*—Artificial dielectrics, backward waves, focusing, left-handed media (LHM), metamaterials, periodic structures.**

I. Introduction

RECENTLY, synthesized artificial dielectric media (metamaterials) have been demonstrated to exhibit negative electric permittivity and magnetic permeability simultaneously over a certain range of frequencies. Consequently, the refractive index of such metamaterials exhibits a negative value in this frequency range. These materials have also been termed left-handed media (LHM) because the electric field $\overline{E}$, the magnetic field $\overline{H}$, and the propagation vector $\overline{k}$ form a left-handed triplet, in contrast to normal (right-handed) media [1]–[6]. This kind of "left handedness" can be interpreted in terms of the ability of these media to support propagating backward waves [7], [8]. As predicted by Veselego [1], interesting new phenomena may be observed when these materials react to an incident electromagnetic field, such as reversed refraction, backward Cherenkov radiation, reversed Doppler effect, as well as near field focusing from homogenous slabs. In recent experiments, such three-dimensional (3-D) media have been constructed by using split-ring resonators (SRR) and thin wires and successfully demonstrated reversed refraction [5]. These SRR/wire media exhibit a negative refractive index around the SRR resonance[2]–[5].

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In this letter, we present a transmission line model to analyze and interpret the SRR/wire metamaterial. During the analysis, it is shown that the SRR resonance is not a necessary condition for achieving a negative refractive index. In fact, it is demonstrated that two-dimensional (2-D) transmission-line networks can be periodically loaded with series capacitors and shunt inductors to obtain a negative refractive index. Such media support propagating backward waves which makes them left handed, thus exhibiting a negative refractive index over a wide bandwidth [7], [8].

II. Transmission Line Model of the SRR/Wire Metamaterial

Transmission line modeling (TLM) can be employed to explain the electromagnetic propagation behavior in media [9]. Homogeneous, isotropic media can be represented in terms of equivalent transmission line (TL) circuits. In such a TL circuit, the electric permittivity (ε) and magnetic permeability (μ) are represented by the distributed series impedance (Z') and shunt admittance (Y') per unit length respectively, according to

$$Z' = j\omega\mu, \qquad Y' = j\omega\varepsilon. \tag{1}$$

The effective propagation constant (γ) is given by

$$\gamma = \sqrt{Z'Y'} = j\omega\sqrt{\mu\varepsilon}. \tag{2}$$

It should be pointed out from the onset that the treatment in this paper is based on one-dimensional (1-D) models but this is not a serious limitation since the same discussion can be readily extended to two or three dimensions.

Consider the effective medium theory for characterizing the SRR/wire left-handed medium proposed by Pendry, Smith, and colleagues [2], [3], [6]. The corresponding effective relative electric permittivity (ε_{eff}) and magnetic permeability (μ_{eff}) of this composite medium are given by [3]

$$\varepsilon_{eff} = 1 - \frac{\omega_p^2}{\omega^2}, \qquad \mu_{eff} = 1 - \frac{F\omega_o^2}{\omega^2 - \omega_o^2 - j\omega\Gamma} \tag{3}$$

where, $f_p = \omega_p/2\pi$ is the plasma frequency of the thin wire, $f_o = \omega_o/2\pi$ is the resonant frequency of the SRR, F is a factor that controls the bandwidth of the negative refractive index and represents the fractional area occupied by the split-ring resonator in the unit cell, and Γ is the dissipation factor which is determined by the conductor loss in the SRR.

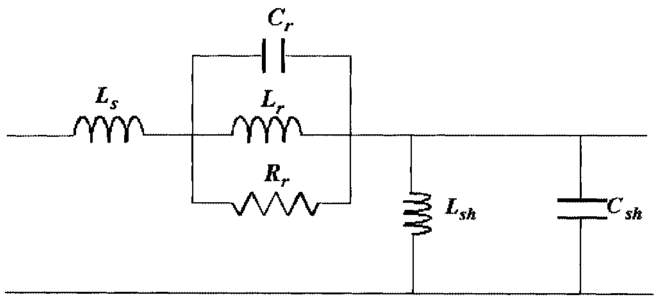

Fig. 1. Unit cell of the equivalent transmission line circuit of the SRR/wire medium.

Fig. 1 shows the proposed TL circuit representing a unit cell of length d for the SRR/wire medium as described by (3), where d is small compared to the wavelength. The series branch corresponds to the SRR whereas the shunt branch represents the thin wire. The unit cell parameters (series impedance Z and shunt admittance Y) can be translated to their distributed counterparts (Z' and Y') by using the transformation: $Z' = Z/d$ and $Y' = Y/d$. Using (2), the propagation constant γ of the transmission line model can be written as

$$\gamma_t = \frac{j\omega}{c} \cdot \sqrt{\left(\left(\frac{L_s}{\mu_o d}\right) - \frac{\frac{L_r}{\mu_o d}\,\frac{1}{L_r C_r}}{\omega^2 - \frac{1}{L_r C_r} - \frac{j\omega}{R_r C_r}}\right)\left(\left(\frac{C_{sh}}{\varepsilon_o d}\right) - \frac{1}{\varepsilon_o \omega^2 L_{sh} d}\right)}. \tag{4}$$

Here, $c = 1/\sqrt{\mu_o \varepsilon_o}$. The term under the square root corresponds to the product of μ_{eff} and ε_{eff}. Comparing (4) with (3), we find that for the following values of the lumped element values, the unit cell shown in Fig. 1 is analogous to that of the SRR/wire medium

$$L_s = \mu_o d, \qquad L_r = (\mu_o d)F, \qquad C_r = \frac{1}{\omega_o^2 (\mu_o d) F},$$
$$R_r = \frac{1}{\Gamma C_r}, \qquad C_{sh} = \varepsilon_o d, \qquad L_{sh} = \frac{1}{\omega_p^2 (\varepsilon_o d)}. \tag{5}$$

As shown in Fig. 1, the unit cell of the proposed transmission line model for the SRR/wire medium consists of three resonators, two on the series branch and one on the shunt branch. The series branch consists of two resonators. The first resonator is the parallel $R_r L_r C_r$ one which has a resonant frequency of f_o. The second resonator is formed by the combination of the parallel $R_r L_r C_r$ resonator and the series inductor L_s and it resonates at $f_b = f_o\sqrt{1+F}$. On the other hand, the shunt branch is a parallel resonator $L_{sh} C_{sh}$ with resonance at f_p. The resonant frequencies of the SRR/wire medium and the remaining parameters in (3) can now be defined in terms of the unit cell parameters as follows:

$$f_o = \frac{1}{2\pi\sqrt{L_r C_r}} \qquad f_b = \frac{1}{2\pi\sqrt{L_r C_r}}\sqrt{1 + \frac{L_r}{L_s}},$$
$$f_p = \frac{1}{2\pi\sqrt{L_{sh} C_{sh}}} \; F = \frac{L_r}{L_s} \qquad \Gamma = \frac{1}{R_r C_r}. \tag{6}$$

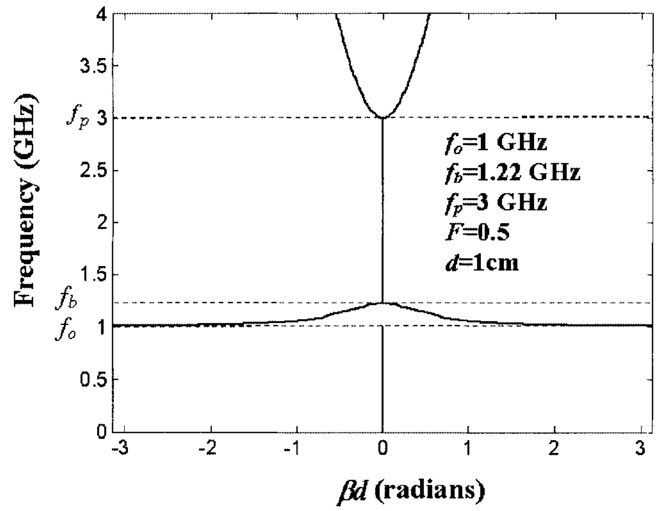

Fig. 2. Typical dispersion diagram for the transmission line model of the SRR/wire medium (d = 1 cm).

III. TL Analysis of the Dispersion Characteristics of the SRR/Wire Medium

Fig. 2 shows a typical dispersion curve for the SRR/wire medium. Here conductor losses are ignored ($\Gamma = 0$ or $R_r = \infty$) for simplicity. The medium exhibits passbands with a positive refractive index when μ_{eff} and ε_{eff} are simultaneously positive. Furthermore, it exhibits passbands with a negative refractive index when both μ_{eff} and ε_{eff} are negative.

From the transmission-line perspective, the signs of the series and shunt reactances should be opposite. In other words, if one is inductive the other should be capacitive to obtain a passband. Below the cut-off frequency f_o of the parallel resonator $L_r C_r$, both the series and shunt branches are inductive. Hence there is no propagation. Between f_o and f_b, the series branch is capacitive whereas the shunt branch is inductive. In this region, energy propagation takes place by backward waves for which the phase velocity is opposite to the energy velocity (or group velocity). Alternatively, this is the region where the propagation $\overline{k}$ vector points in a direction opposite to the Poynting $\overline{S}$ vector. These propagating backward waves imply a negative refractive index [7]. Moreover, above the cut-off frequency of the second resonator f_b, the series branch becomes inductive again and hence the medium exhibits another stopband between f_b and f_p. Finally, the shunt branch becomes capacitive above the third cut-off frequency f_p of the shunt resonator $L_{sh} C_{sh}$, and the transmission line exhibits a passband. However, since the series branch is inductive and the shunt branch is capacitive, phase and group velocities are both positive. Equivalently in the SRR/wire medium, $\overline{k}$ and $\overline{S}$ point to the same direction, thus leading to a positive refractive index.

IV. Loaded Left-Handed Transmission Line Metamaterials Without Excess Resonators

From the previous discussion, it becomes clear that the condition for obtaining a negative refractive index in the transmission line model of Fig. 1 is satisfied when the series branch is capacitive and the shunt branch is inductive. Indeed, this implies

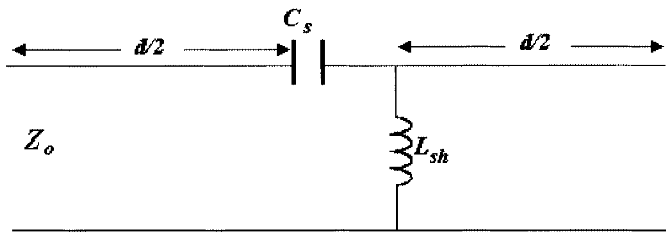

Fig. 3. Unit cell of a transmission line loaded with a series capacitor and a shunt inductor.

the propagation of backward waves with negative phase velocity and positive group velocity. Thus in Fig. 1 it is not necessary to include the parallel (SRR) resonator in the series branch, a single series capacitor suffices. Implementing this idea based on printed transmission-line networks, compact, fully planar, broadband negative refractive index media have been proposed [7], [8]. Fig. 3 shows the unit cell of such a resonator-free L-C loaded transmission line medium. If Z_o is the characteristic impedance of the unloaded line and the length of the transmission line segment d per unit cell is assumed to be very small compared to the wavelength, the propagation constant for $kd \ll 1$ and $\beta d \ll 1$ can be approximated as

$$\beta \cong \omega\sqrt{L_o' C_o'}\sqrt{\left(1-\frac{1}{\omega^2 (L_{sh} d) C_o'}\right)\left(1-\frac{1}{\omega^2 L_o' (C_s d)}\right)} \quad (7)$$

where L_o' and C_o' are the distributed inductance and capacitance per unit length of the underlying (unloaded) transmission line. The various cut-off frequencies can be defined in terms of the transmission line parameters as follows:

$$f_o' = \frac{1}{4\pi\sqrt{L_{sh}C_s}}, \quad f_b' = \frac{1}{2\pi\sqrt{L_o C_s}}, \quad f_p' = \frac{1}{2\pi\sqrt{L_{sh}C_o}} \quad (8)$$

where $L_o = L_o' d$ and $C_o = C_o' d$ are the unit cell inductance and capacitance of the transmission line. Evidently, these series-C/shunt-L loaded transmission line structures offer great flexibility in tailoring the negative refractive index frequency bandwidth by varying the loading elements, for fixed d. On the other hand, the bandwidth may be made arbitrarily large by decreasing the periodicity d. Such flexibility is lagging from the SRR/wire media which operate around (above) the resonance of the SRRs and the negative refractive index bandwidth is restricted by the fractional area occupied by the SRR and characterized by the parameter $0 < F < 1$.

It is worth mentioning that the structure of Fig. 3 is well known and has been studied extensively in the context of 1-D backward-wave propagation [10]. However, what is striking here is that 2-D versions of such periodically loaded transmission line media lend themselves as practical left-handed media that are capable of demonstrating reversed refraction and focusing when interfaced with conventional right-handed media such as a parallel-plate waveguide [7],[10].

V. Conclusion

We have proposed a transmission line equivalent circuit model to analyze the propagation characteristics of left-handed SRR/wire metamaterials. In this context, such media exhibit a negative refractive index when the series branch of the equivalent transmission line model is capacitive and the shunt branch is inductive. Therefore, it becomes clear that one can directly implement left-handed media using L–C loaded transmission lines without the need for a SRR resonator. For example, the negative refractive index condition can be satisfied by loading a host transmission line network with series capacitors and shunt inductors. The left-handed transmission line media thus synthesized can offer large operating bandwidths and naturally lend themselves for completely planar implementations which is desirable for RF/microwave devices and circuits.

References

[1] V. G. Veselego, "Electrodynamics of substances with simultaneously negative electrical and magnetic permeabilities," *Sov. Phys. Usp.*, vol. 10, no. 4, pp. 509–514, Jan.–Feb. 1968.

[2] D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, "Composite medium with simultaneously negative permeability and permittivity," *Phys. Rev. Lett.*, vol. 84, no. 18, pp. 4184–4187, May 2000.

[3] D. R. Smith and N. Kroll, "Negative refractive index in left-handed materials," *Phys. Rev. Lett.*, vol. 85, no. 14, pp. 2933–2936, Oct. 2000.

[4] R. A. Shelby, D. R. Smith, S. C. Nemat-Nasser, and S. Schultz, "Microwave transmission through a two-dimensional, isotropic, left-handed metamaterial," *Appl. Phys. Lett.*, vol. 78, no. 4, pp. 489–491, Jan. 2001.

[5] R. A. Shelby, D. R. Smith, and S. Schultz, "Experimental verification of a negative index of refraction," *Science*, vol. 292, no. 4, pp. 77–79, Apr. 2001.

[6] J. B. Pendry, A. J. Holden, D. J. Robins, and W. J. Stewart, "Magnetism from conductors and enhanced nonlinear phenomena," *IEEE Trans. Microwave Theory Tech.*, vol. 47, pp. 2075–2084, Nov. 1999.

[7] A. K. Iyer and G. V. Eleftheriades, "Negative refractive index metamaterials supporting 2-D waves," in *IEEE MTT-S Symp. Dig.*, vol. 2, Seattle, WA, June 2–7, 2002, pp. 1067–1070.

[8] A. Grbic and G. V. Eleftheriades, "Experimental verification of backward-wave radiation from a negative refractive index metamaterial," *J. Appl. Phys.*, vol. 92, no. 10, July 2002.

[9] C. Christopoulos, *The Transmission-Line Modeling Method: TLM*. Piscataway, NJ: IEEE, 1995.

[10] S. Ramo, J. R. Whinnery, and T. Van Duzer, *Fields and Waves in Communication Electronics*, 3rd ed. New York: Wiley, 1994, pp. 263–264.